

# Chimeras in globally coupled oscillatory systems: From ensembles of oscillators to spatially continuous media

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We study an oscillatory medium with a nonlinear global coupling that gives rise to a harmonic mean-field oscillation with constant amplitude and frequency. Two types of cluster states are found, each undergoing a symmetry-breaking transition towards a related chimera state. We demonstrate that the diffusional coupling is non-essential for these complex dynamics. Furthermore, we investigate localized turbulence and discuss whether it can be categorized as a chimera state.

An oscillatory medium may exhibit a variety of spatio-temporal patterns. A cluster pattern consisting of distinct uniformly oscillating regions with mutual phase shifts is a prominent example. Surprisingly, the symmetry of this state can be broken and a so-called chimera state may form. Then, some of the regions display turbulent dynamics, while the other regions remain synchronized. This coexistence of synchrony and incoherence arouse much interest and triggered many investigations. Unihemispherical sleep, performed by, e.g., various dolphins and birds, may be connected to the phenomenon of chimera states, since during this kind of sleep one cerebral hemisphere is awake (desynchronized), while the other one is sleeping (synchronized).

The prerequisites for such chimera states are not completely known yet. For globally coupled systems it seems that a clustering mechanism is needed as a first symmetry-breaking step. In this Article, we investigate two different kinds of cluster states that undergo each a transition to a corresponding chimera state. The chimera states then share properties with the cluster states, from which they originate. Furthermore, we discuss the role of the diffusional coupling and point out that in many cases it is sufficient to consider only the global coupling in order to understand the complex dynamics. However, so-called localized turbulence seems to rely on diffusion. This state is similar to chimera states, while the lack of a phase boundary between synchrony and incoherence renders its detailed dynamics different.

## I. INTRODUCTION

Oscillatory media with a global coupling display a wealth of spatio-temporal dynamics. Considering the medium as a field of diffusively coupled oscillators<sup>1</sup>, each oscillator experiences exactly the same force from the global coupling. Counterintuitively, this can lead to a bistability of the dynamics, i.e., there exist two stable stationary states and each oscillator can settle to one of them. Therefore, cluster formation may be observed. In a cluster state, the system separates into distinct regions with mutual phase shifts. In several experimental setups a global coupling arises naturally or can be implemented easily and thus, cluster formation has been observed in various systems<sup>2-6</sup>.

Astonishingly, the symmetry of the cluster state can be broken in such a way that one observes the coexistence of synchronously oscillating regions and regions displaying turbulence. This state has been termed a chimera state<sup>7</sup>, referring to the chimera in Greek mythology, a hybrid creature consisting of parts of different animals. Chimera states have been found in many different settings. Most of the theoretical investigations deal with phase models with a nonlocal coupling scheme, i.e., the coupling strength decreases with the distance between two oscillators<sup>7-9</sup>. Furthermore, discrete systems like coupled map lattices, may exhibit chimera states. Finally, chimera states have been demonstrated to exist in several experiments. For a recent review, see Ref.<sup>9</sup>.

The photoelectrodissolution of n-type silicon is one of the experimental systems exhibiting chimera states. Unlike in most settings, in this system the chimera states emerge spontaneously, without external feedback or specifically prepared initial conditions<sup>10,11</sup>. On the silicon working electrode an oxide layer forms and the thickness of this layer features the spatio-temporal patterns. We modeled this continuous medium successfully employing a complex Ginzburg-Landau equation with a nonlinear global coupling<sup>10,12</sup>. Omitting the diffusional coupling in the equation and thus, studying an ensemble of Stuart-Landau oscillators with nonlinear global coupling, we were able to demonstrate that the chimera

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states we found form under solely global coupling<sup>10</sup>. A more detailed investigation of chimera states in the Stuart-Landau ensemble - in fact, we found two types of chimeras - revealed that for globally coupled systems a clustering mechanism and non-isochronicity of oscillators is sufficient for chimera states to exist<sup>13</sup>. Non-isochronicity means that the frequency of oscillation depends on the actual amplitude. These conclusions are in accordance with other studies on chimera states under global coupling<sup>14,15</sup>.

In this Article, we bring continuous media and ensembles of oscillators closer together by demonstrating that both types of chimera states found in the Stuart-Landau ensemble with nonlinear global coupling are present in the modified complex Ginzburg-Landau equation (MC-GLE), the model for the photoelectrodissolution of n-type silicon. More precisely, we present two types of cluster states giving rise to two types of chimera states and connect them with the corresponding states in the discrete ensemble of oscillators. It turns out that the mechanisms discussed for the discrete system are also valid in the continuous model. Especially, the chimera state found in the ensemble under linear global coupling<sup>16</sup> is also present in the corresponding CGLE with linear global coupling. Furthermore, we investigate also so-called localized turbulence, for which diffusion seems to be essential, and discuss whether this state should also be classified as a chimera state.

## II. CGLE WITH NONLINEAR GLOBAL COUPLING

In the photoelectrodissolution of n-type silicon, an oxide-layer forms at the Si|electrolyte interface. Its thickness is determined by the interplay of electrooxidation and an etching process and can exhibit sustained oscillations<sup>6,17</sup>. In fact, these oscillations originate in a Hopf bifurcation, as shown in Refs.<sup>12,18</sup>. When spatially resolving the oxide-layer thickness, one observes that it can also exhibit spatio-temporal pattern formation with a huge variety of different dynamical states<sup>6,11</sup>. Astonishingly, for most of the patterns, the spatially averaged oxide-layer thickness oscillates almost harmonically. To model these dynamics, i.e., a system near a supercritical Hopf bifurcation with conserved harmonic mean-field oscillations, in a general way, we use a complex Ginzburg-Landau equation with an additional nonlinear global coupling<sup>6,10,12,19</sup>:

$$\partial_t W = W + (1 + ic_1) \nabla^2 W - (1 + ic_2) |W|^2 W - (1 + i\nu) \langle W \rangle + (1 + ic_2) \langle |W|^2 W \rangle. \quad (1)$$

$W$  describes a complex two-dimensional field  $W = W(x, y, t)$ , where  $x$  and  $y$  are spatial coordinates and  $t$  denotes time.  $\langle \dots \rangle$  stands for spatial averages. Calculating the spatial average of the whole equation, we obtain for the mean-field

$$\partial_t \langle W \rangle = -i\nu \langle W \rangle \Rightarrow \langle W \rangle = \eta e^{-i\nu t}. \quad (2)$$

Thus, we indeed describe a two-dimensional system in the vicinity of a Hopf bifurcation with conserved harmonic mean-field oscillation. We solve Eq. (1) numerically as described in the Appendix. In the next two sections we examine two types of cluster patterns undergoing each a symmetry-breaking transition towards a chimera state.

## III. TYPE I DYNAMICS

### A. Nonlinear global coupling

The first type of dynamics we present are patterns related to well-known amplitude clusters<sup>20</sup>. The dynamics of such amplitude clusters are visualized in Fig. 1a in a two-dimensional snapshot showing  $|W(x, y)|$  (left), a one dimensional cut as indicated in the snapshot versus time (middle) and a snapshot of the arrangement of the local oscillators in the complex plane (right). Parameters read  $c_1 = 0.2$ ,  $c_2 = 0.56$ ,  $\nu = 1.5$  and  $\eta = 0.9$ . Amplitude clusters consist of regions separated mainly by an amplitude difference. Both groups oscillate, with a small phase difference to each other, at constant, but different amplitude.

Changing parameter  $c_2$  to  $c_2 = 0.58$ , one of the cluster regions undergoes a symmetry-breaking transition, as shown in Fig. 1b. Synchronized regions of constant amplitude coexist with regions, where amplitude waves are emitted from the boundaries and from amplitude-spiral cores. The amplitude spirals are visible in the two-dimensional snapshot. Interestingly, there is no amplitude defect in the spiral core and the spiral dynamics take place in a very curved and confined region. Here, dynamics are mainly found in the modulus, while the phase hardly exhibits any spatial pattern and is more or less uniform in the two regions, with a small phase shift between the regions. This is also obvious from the arrangement in the complex plane. In the amplitude cluster state in Fig. 1a, the two groups of oscillators reside at the two ends of the string visible in the complex plane. Going to spiral-wave like dynamics in Fig. 1b, the group at the lower radius spreads and thus forms a bunch. Inside the bunch the relative amplitude differences are much larger than the mutual phase differences.

This bunch of oscillators at the lower radius desynchronizes strongly when changing parameters to  $c_2 = 0.61$  and  $\eta = 1.0$ . Now, the symmetry-breaking is more dramatic: Synchronized regions of constant amplitude coexist with regions displaying amplitude turbulence, see Fig. 1c. This is an example for a chimera state, which is mainly present in the modulus of the dynamics. Dynamics in the desynchronized regions are reminiscent of intermittent behavior in the standard complex Ginzburg-Landau equation<sup>21</sup>: homogeneous spots pop up in an irregular manner, vanishing slowly afterwards. Amplitude defects do not occur. One observes a slight oscillation in the amplitude of the homogeneous part and this seems to be connected to variations in the spatial extension of the

desynchronized regions, as visible in the one-dimensional cut. Furthermore, in both, the spiral-wave like dynamics and the chimera state, the dynamics in the incoherent regions are overlayed by an overall oscillation, washing out the patterns repeatedly, as visible, e.g., at the end of the cut shown in Fig. 1b. However, it does not seem that the dynamics are resetted, but rather the visualization becomes blurry.

Type I chimeras are related to amplitude clusters, as the two groups in both states are separated by an amplitude difference. Thus, the clustering mechanism is needed to yield two separated groups in order to obtain the dynamics presented here. Then, the symmetry is broken due to nonlinear amplitude effects: since the response on a force depends on the amplitude of the oscillator, the response is different in the two groups at different moduli. As we will see in Section IV, the modulated amplitude clusters give rise to a second type of chimera states.

## B. Linear global coupling

Interestingly, one finds type I chimeras also in a CGLE with linear global coupling:

$$\begin{aligned} \frac{\partial W}{\partial t} = & W - (1 + ic_2) |W|^2 W + (1 + ic_1) \nabla^2 W \\ & + K(1 + ic_3) (\langle W \rangle - W) . \end{aligned} \quad (3)$$

The linear average  $\langle W \rangle$  constitutes the linear global coupling,  $K(1 + ic_3)$  is the complex coupling strength with parameters  $K$  and  $c_3$ . Numerical results are depicted in Fig. 2. The one-dimensional cut of  $|W|$  reveals that the spatio-temporal dynamics are qualitatively the same as in the type I chimeras found in the MCGLE, see Fig. 1c. Moreover, the arrangement of local oscillators in the complex plane in Fig. 2c resembles the configuration in case of the MCGLE. Differences are that the incoherent domains in Fig. 2a are not of oval shape and that one observes a rather irregular oscillation in the modulus in the one-dimensional cut, instead of a more periodic one. These states are related to chimera states found in an ensemble of Stuart-Landau oscillators with linear global coupling<sup>16</sup>. We described in Ref.<sup>13</sup> that in case of the chimera state under linear global coupling the mean-field oscillates approximately harmonically and that the amplitude clusters and type I chimeras found in our model under nonlinear global coupling constitute idealized dynamics of the corresponding patterns found under linear global coupling.

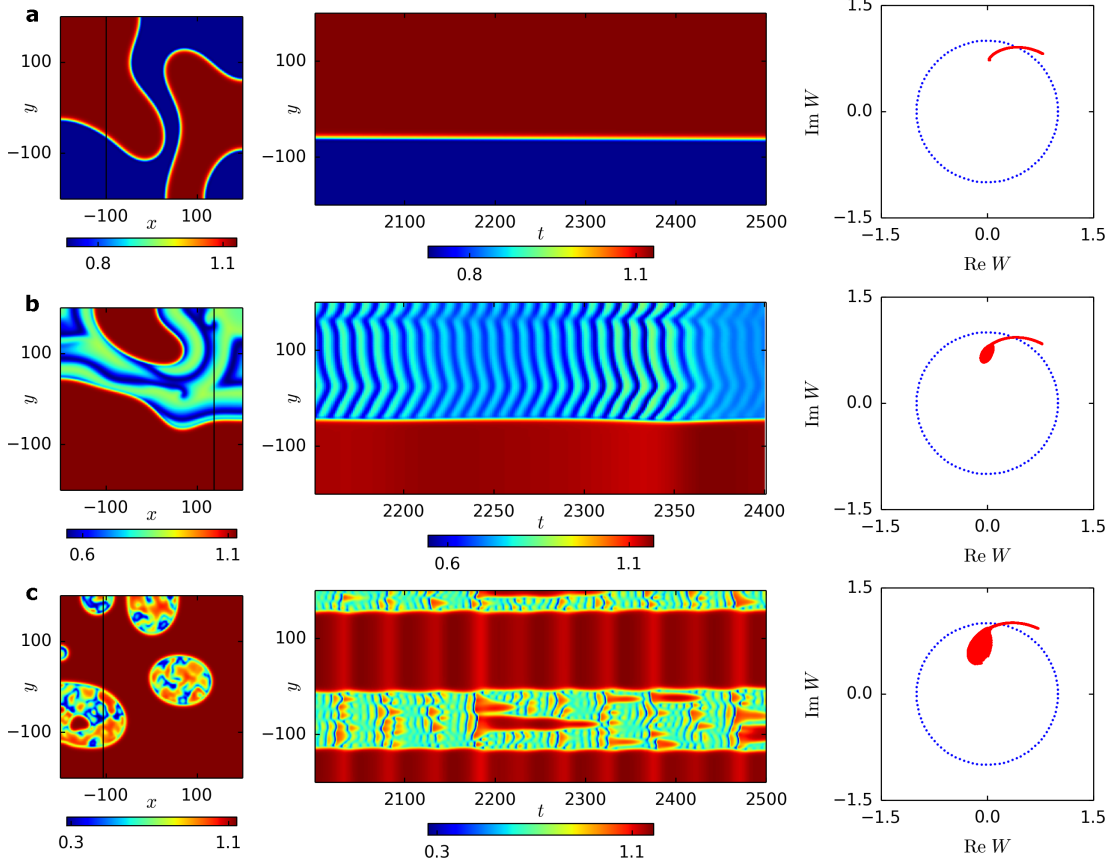


FIG. 1. Type I patterns found in Eq. (1): snapshots of  $|W|$  (left column), one-dimensional cuts versus time (also  $|W|$ ) as indicated by the vertical lines in the snapshots (middle column) and snapshots of the arrangement of local oscillators in the complex plane (right column). (a) Amplitude clusters ( $c_1 = 0.2$ ,  $c_2 = 0.56$ ,  $\nu = 1.5$ ,  $\eta = 0.9$ ). (b) Coexistence of synchrony and spiral-wave like dynamics. Waves are emitted from the region boundaries and from the spiral cores ( $c_1 = 0.2$ ,  $c_2 = 0.58$ ,  $\nu = 1.5$ ,  $\eta = 0.9$ ). (c) Type I chimera state consisting of oval-shaped regions exhibiting desynchronized behavior in an otherwise homogeneous background ( $c_1 = 0.2$ ,  $c_2 = 0.61$ ,  $\nu = 1.5$ ,  $\eta = 1.0$ ).

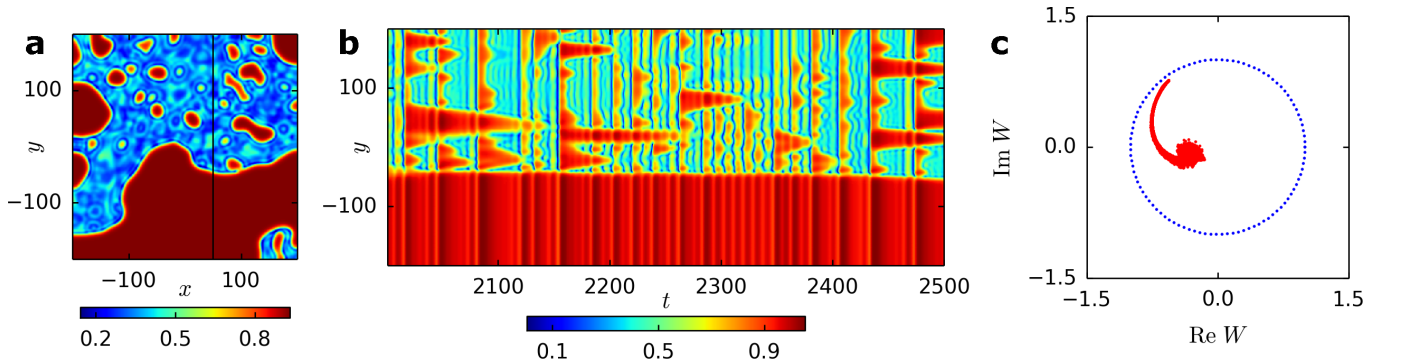


FIG. 2. Type I chimera state in a CGLE with linear global coupling, given in Eq. (3). Shown are a two-dimensional snapshot of the modulus (a), a one-dimensional cut of the modulus versus time (b) and the arrangement of local oscillators in the complex plane (c). Parameters read  $c_1 = 1.2$ ,  $c_2 = 1.7$ ,  $K = 0.67$  and  $c_3 = -1.25$ .

#### IV. TYPE II DYNAMICS

Going back to the dynamics of Eq. (1), for  $c_2 = -0.7$ ,  $\nu = 0.1$  and  $\eta = 0.66$ , we find a second type of cluster pattern, which is presented in Fig. 3a. Note that in the snapshots we plot the real part of  $W$ , as both the modulus and the phase exhibit significant variations in case of the type II dynamics. The system again splits into two phases, but the one-dimensional cut shows that the dynamics are more complex than in case of amplitude clusters. The overall uniform oscillation is modulated by two-phase clusters. These modulational oscillations are visible in the one-dimensional cut in Fig. 3a. Thus, we call this type of clustering modulated amplitude clusters, as the clusters form in the modulational oscillations of the amplitude. This pattern is one of the most prominent patterns in the photoelectrodissolution of n-type silicon<sup>6,10,11,19</sup> and the MCGLE thus reproduces this pattern very well.

The second type of clusters also undergoes a symmetry-breaking transition<sup>10</sup>, here when changing  $c_2$  to  $c_2 = -0.67$ , resulting in subclustering as shown in Fig. 3b. One phase exhibits two phase clusters as a substructure, while the other one stays homogeneous. The substructure-clusters oscillate at half the frequency of the modulational oscillation. Therefore, we suspect the subclustering being connected to a period-doubling bifurcation.

Changing the parameter further to  $c_2 = -0.58$ , the symmetry-breaking becomes again more dramatic: the beforehand existing two-phase subclusters turn into turbulence, thus realizing a second type of chimera state, see Fig. 3c. Again we have to emphasize the connection to the photoelectrodissolution of silicon, since this second type of chimera states could be observed in experiments, too<sup>10,11</sup>. We compared the simulations and the experiments in detail in Refs.<sup>10</sup> and<sup>12</sup>. As discussed in Ref.<sup>10</sup> the hierarchy of patterns is similar to the situation encountered in an experimental realization of a system composed of two distinct groups with different intra- and inter-group coupling<sup>22</sup>. In our case, the two groups, i.e., the dynamically different phases, are created by the clustering mechanism and a difference between intra- and inter-group coupling can be attributed to the phase difference between the groups.

In order to gain more insight into the spatio-temporal dynamics of the type II chimera state, we analyze its frequency spectrum in a spatially resolved manner. Therefore, we perform a Fourier transformation in time of the real part of  $W(\mathbf{r}, t)$  at every point  $\mathbf{r} = (x, y)$ . We spatially average the resulting squared amplitudes  $|a(\mathbf{r}, \omega)|^2$  to obtain the spatially averaged power spectrum  $S(\omega) = \langle |a(\mathbf{r}, \omega)|^2 \rangle$ . The resulting spectrum for the simulation presented in Fig. 3c is depicted in Fig. 4a. Due to the turbulence in the incoherent phase it consists of a large background, but exhibits also two major peaks, marked with circles. The highest peak is at the frequency  $\nu$  of

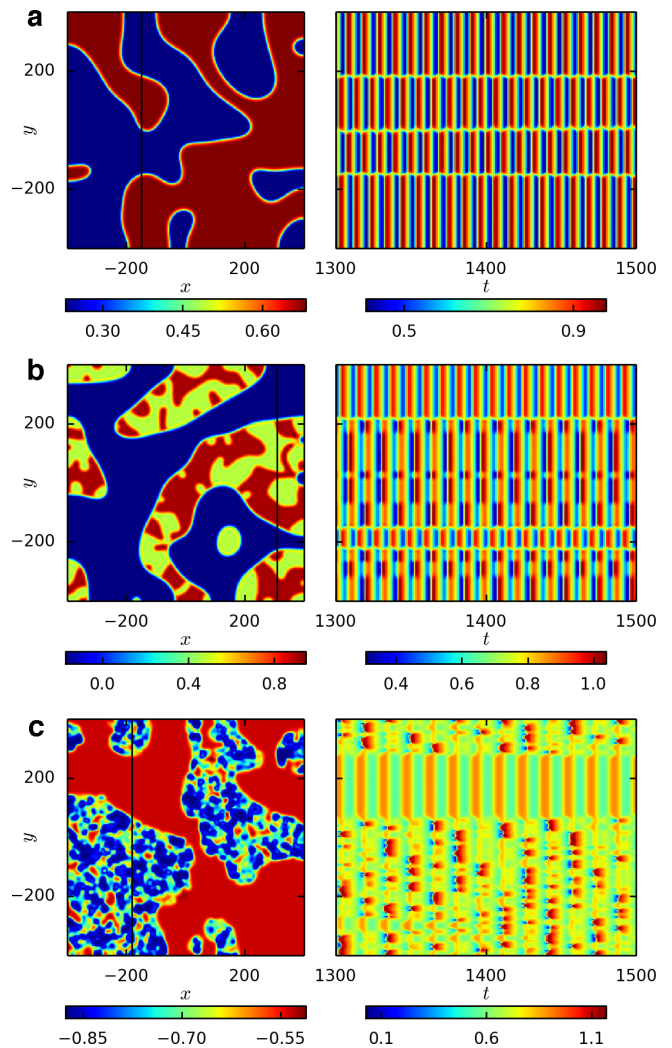


FIG. 3. Type II patterns found in Eq. (1): two-dimensional snapshots in left column ( $\text{Re } W$ ), one-dimensional cuts in right column ( $|W|$ ). (a) Modulated amplitude clusters ( $c_1 = 0.2$ ,  $c_2 = -0.7$ ,  $\nu = 0.1$ ,  $\eta = 0.66$ ). (b) Subclustering, where one phase is synchronized, while the other one exhibits two-phase clusters as a substructure ( $c_1 = 0.2$ ,  $c_2 = -0.67$ ,  $\nu = 0.1$ ,  $\eta = 0.66$ ). (c) Type II chimera ( $c_1 = 0.2$ ,  $c_2 = -0.58$ ,  $\nu = 0.1$ ,  $\eta = 0.66$ ).

the mean-field oscillation, see Eq. (2). The second highest peak stems from the clustering frequency: as outlined above, in the modulated amplitude clusters one observes two major frequencies, one of the homogeneous oscillation and one as a result of the modulational oscillation. In the type II chimera, the separation into two different phases occurs via this clustering mechanism and therefore, some properties of it are still present. This becomes more clear, when inspecting the Fourier amplitudes  $a(\mathbf{r}, \omega)$  at the two highest peaks, which consist of moduli and phases. The distribution of phases of the local oscillators at the two peaks is shown in histograms in Figs. 4b and c. At frequency  $\nu$ , Fig. 4b, the existence of only one, sharp peak demonstrates that all local oscilla-

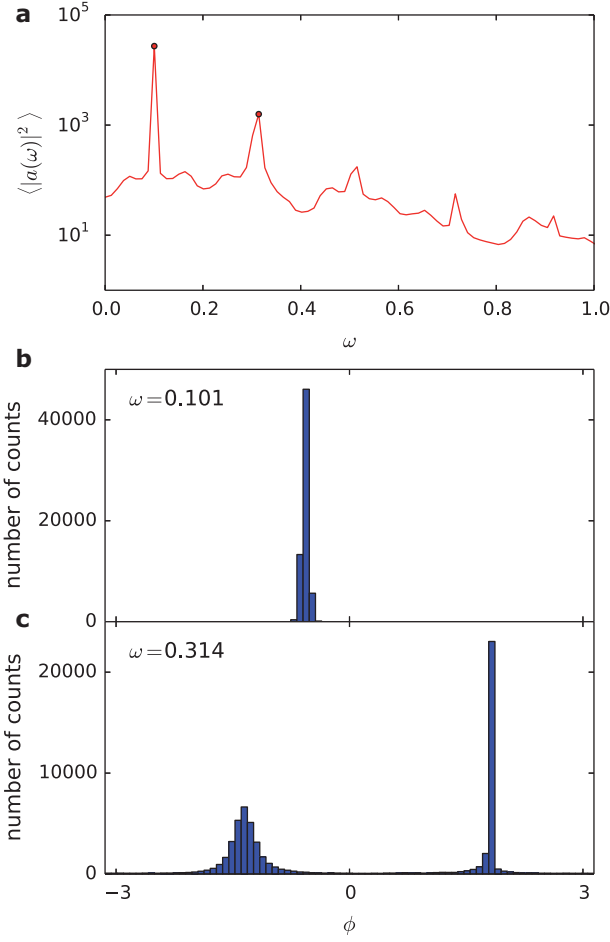


FIG. 4. Spatially averaged power spectrum in (a) and phase histograms for the Fourier amplitudes  $a(\mathbf{r}, \omega)$  at the highest peak in (b) and at the second highest peak in (c), which are indicated by circles in (a).

tors perform the homogeneous oscillation in synchrony. The histogram for the second highest peak at  $\omega \approx 0.31$  (Fig. 4c) reveals that the clustering mechanism is still active: we encounter two peaks for the two groups, which are phase shifted by  $\pi$ , indicating that at this frequency the clustering occurs. The right peak is sharp and describes the synchronized oscillators, while the left peak has a Gaussian-like shape that stems from the incoherent oscillators.

In essence, we encountered two types of cluster states giving rise to two types of chimera states. A better understanding of these dynamics can be obtained when omitting the diffusional coupling, thereby reducing the complexity of the system. In the next section, we will do so and demonstrate that diffusion is non-essential for the dynamics presented so far.

## V. DISPENSABILITY OF DIFFUSION FOR TYPE I AND II DYNAMICS

The spatial coupling in the MCGLE, Eq. (1), is given by the local diffusional coupling and the nonlinear global coupling. Here, we compare the cluster and chimera states of the MCGLE with dynamics found in an ensemble of oscillators subject to nonlinear global coupling only<sup>13</sup>. Therefore, we omit the diffusional coupling in the MCGLE and consider an ensemble of Stuart-Landau oscillators with the nonlinear global coupling:

$$\frac{dW_k}{dt} = W_k - (1 + ic_2)|W_k|^2 W_k - (1 + i\nu) \langle W \rangle_\Sigma + (1 + ic_2) \langle |W|^2 W \rangle_\Sigma. \quad (4)$$

Here,  $k = 1, 2, \dots, N$  and  $\langle \dots \rangle_\Sigma$  denotes the ensemble average, i.e.,  $\langle W \rangle_\Sigma = \sum_{k=1}^N W_k / N$ . We compare the dynamics of oscillators in this Stuart-Landau ensemble with the dynamics of individual oscillators, or points in space, in the extended model, the MCGLE. Therefore, we choose a few points in space in each phase of the spatio-temporal patterns. For the cluster dynamics the resulting time series are depicted in Fig. 5. The evolution in the complex plane in case of amplitude clusters in the MCGLE (a) is indistinguishable from the corresponding dynamics of oscillators in the Stuart-Landau ensemble (b). The same holds for modulated amplitude clusters in the MCGLE (c) and in the ensemble (d). A similar conclusion could be made for cluster patterns during the CO oxidation on Pt(110): Falcke & Engel demonstrate that clusters arise with the global coupling alone, no local coupling is needed<sup>23</sup>. In their experiment, the local coupling is due to surface diffusion of mobile adsorbates and the global coupling is due to pressure changes in the gas phase.

In Fig. 6 the temporal dynamics in the two types of chimera states are considered. Individual oscillators in the MCGLE in case of type I chimeras (a) exhibit the same qualitative features as oscillators in the discrete ensemble (b): the synchronized group has a larger modulus, while the incoherent oscillators show irregular motion at lower values of the modulus. Note that the  $c_2$ ,  $\nu$  and  $\eta$  values differ a bit between the extended and the discrete model. This is attributable to the change of stability by the diffusional coupling, i.e., in the extended system the chimera state spontaneously emerges at other parameter values as the diffusional coupling shifts the stability of the solutions. This might also be the reason why type I chimeras seem to be stable in the MCGLE, while we could only find unstable type I chimeras (undergoing heteroclinic transitions) in the Stuart-Landau ensemble<sup>13</sup>. However, on a qualitative basis the discrete model describes the same type I chimeras as the MCGLE.

This holds also for type II chimeras, which are compared in Figs. 6c and d, for the MCGLE and the Stuart-Landau ensemble, respectively. In the MCGLE the oscillation of the synchronized group is not as harmonic as in



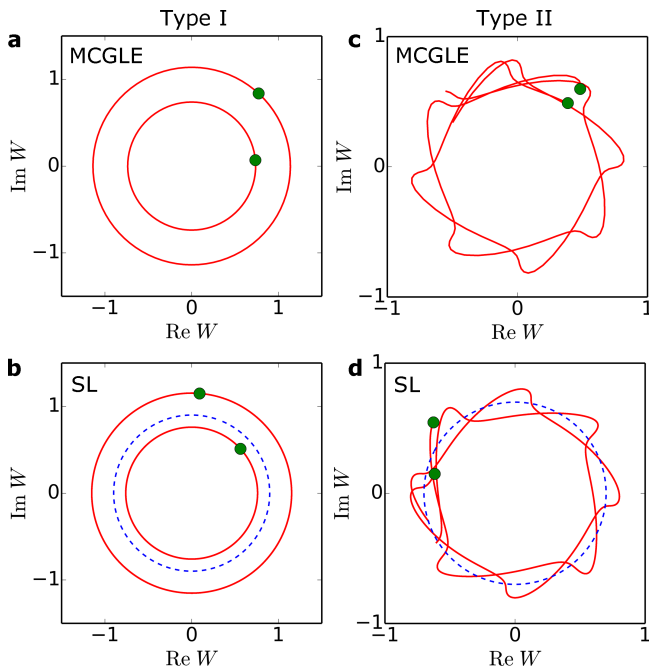


FIG. 5. Cluster dynamics of individual oscillators in the MCGLE and in the Stuart-Landau ensemble (SL). (a) Amplitude cluster in the MCGLE ( $c_1 = 0.2$ ,  $c_2 = -0.56$ ,  $\nu = -1.5$ ,  $\eta = 0.9$ ). (b) Amplitude cluster in the Stuart-Landau ensemble ( $c_2 = -0.56$ ,  $\nu = -1.5$ ,  $\eta = 0.9$ ). (c) Modulated amplitude cluster in the MCGLE ( $c_1 = 0.2$ ,  $c_2 = -0.6$ ,  $\nu = 0.1$ ,  $\eta = 0.7$ ). (d) Modulated amplitude cluster in the Stuart-Landau ensemble ( $c_2 = -0.6$ ,  $\nu = 0.1$ ,  $\eta = 0.7$ ). In (b,d) the dashed line describes the mean-field. Reprinted (d) with permission from [L. Schmidt & K. Krischer, *Phys. Rev. E* **90**, 042911 (2014)]. Copyright (2014) by the American Physical Society (DOI: 10.1103/PhysRevE.90.042911).

the discrete model; the modulational oscillations are far more pronounced. The reason for this difference is that in the discrete model the desynchronized group is much smaller than the synchronized group, while in the extended system we observe approximately phase balance, i.e., both groups are of the same size. In both cases the mean-field has to oscillate harmonically. Hence, the spiking of the incoherent oscillators has to be compensated by the synchronized group, which is more pronounced if the groups are of the same size. Furthermore, due to the diffusional interaction of nearby oscillators, the spiking is smoother in the MCGLE. Again, the parameters are a bit different for the same reason as in case of type I chimeras.

Besides minor differences in the details of the dynamics, we can conclude that also the dynamics in the two types of chimera states in the MCGLE are sufficiently captured by the discrete Stuart-Landau ensemble.

In the MCGLE, a bifurcation analysis of the cluster states is very involved due to the high number of degrees of freedom. With the knowledge that the diffusional coupling is dispensable, the problem highly simplifies. In Ref.<sup>24</sup> we considered two-cluster solutions in the Stuart-

Landau ensemble and performed a bifurcation analysis. It turned out that starting with the synchronized solution the modulated amplitude clusters arise in a secondary Hopf bifurcation, giving rise to a motion on a torus. The two frequencies are then given by  $\nu$  (frequency of the homogeneous mode) and the frequency of the secondary Hopf bifurcation. Amplitude clusters are then created in a saddle-node of infinite period bifurcation, thereby destroying the torus. The two solutions of the amplitude clusters can merge in a pitchfork bifurcation with the synchronized solution. For more details see Ref.<sup>24</sup>.

We can conclude that several complex spatio-temporal dynamics can be described without diffusion. This enables one to analyze the dynamics in more detail. However, this is not true for all solutions of Eq. (1). In the next section we consider spatio-temporal dynamics that rely on a diffusional coupling.

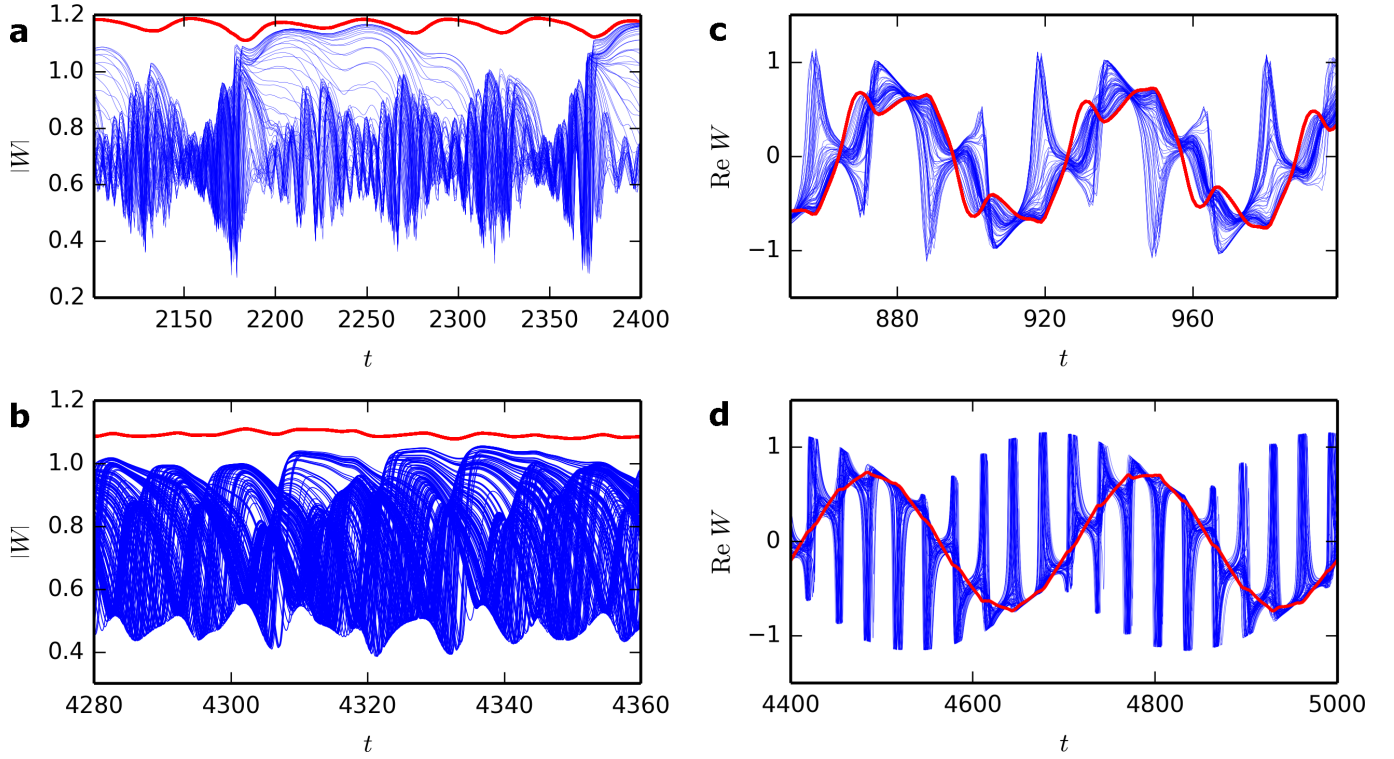


FIG. 6. Dynamics in the two types of chimera states of individual oscillators in the MCGLE and in the Stuart-Landau ensemble. Red denotes the synchronized group and blue the desynchronized one. (a) Type I chimera in the MCGLE ( $c_1 = 0.2$ ,  $c_2 = 0.61$ ,  $\nu = 1.5$ ,  $\eta = 1.0$ ). (b) Type I chimera in the Stuart-Landau ensemble ( $c_2 = 0.58$ ,  $\nu = 1.49$ ,  $\eta = 1.021$ ). (c) Type II chimera in the MCGLE ( $c_1 = 0.2$ ,  $c_2 = -0.58$ ,  $\nu = 0.1$ ,  $\eta = 0.66$ ). (d) Type II chimera in the Stuart-Landau ensemble ( $c_2 = -0.6$ ,  $\nu = 0.021$ ,  $\eta = 0.7$ ).



## VI. LOCALIZED TURBULENCE

For a totally different set of parameters,  $c_1 = -1.6$ ,  $c_2 = 1.5$ ,  $\nu = 1.5$  and  $\eta = 0.9$ , we encounter so-called localized turbulence. We depict our results in Fig. 7, where we present three snapshots at times indicated in the figures in (a), a one-dimensional cut versus time in (b) and a snapshot of the arrangement of local oscillators in the complex plane in (c). Here, we show again  $|W|$ . Small spots of turbulence are randomly created in an otherwise homogeneous background, which then move through the system and spread, but always keeping an overall small size. As for their creation, they also vanish in an irregular manner. The turbulence can be described as circular defect lines (or at least lines of very small amplitude) expanding and breaking down. After their breakdown, new defect circles are created. The mean extent of the turbulent region grows continuously when changing parameters such that we go from a synchronized state to a fully turbulent state. Thus, localized turbulence is a transitional state between synchronous oscillations and turbulence. It has also been observed in the CGLE with delayed global coupling<sup>25</sup>.

Localized turbulence is a phenomenon similar to chimera states as the system also shows the coexistence of synchronized and turbulent regions. However, the separation of these regions is fundamentally different. In the above chimera states and also those discussed in literature, there is a clear separation of coherent and incoherent regions via a well-defined boundary, as clearly visible in Fig. 1c, Fig. 2 and Fig. 3c. Also in nonlocally coupled systems, where the boundaries are rather smooth, the separation is clearly identifiable<sup>8</sup>. Furthermore, the boundaries move only on a slow timescale. In contrast, the incoherent spots in localized turbulence are not separated from the homogeneous regions by phase boundaries. Hence, we face here the situation that although there is a coexistence of synchrony and incoherence, matching formally this most simple definition of a chimera state, the two phases, i.e., the synchronized and the incoherent phases, cannot be clearly identified. In our view, this suggests that localized turbulence should be discriminated from a chimera state. However, this would necessitate a somewhat more restrictive and perhaps more rigorous definition of a chimera state than we have it today.

## VII. CONCLUSIONS

With this study we widened the settings under which chimera states exist: for each type of cluster state we observe, there exists a corresponding chimera state. Since cluster formation has been studied in various experimental setups, one can expect much more experimental chimera states in the future.

We presented numerical results on the dynamics of an oscillatory medium with a nonlinear global coupling that

leads to preserved, harmonic mean-field oscillations. It is a general model that captures the dynamics observed during the photoelectrodissolution of n-type silicon: the type II patterns presented in Section IV reproduce experimentally found spatio-temporal dynamics very well.

Two types of cluster states undergo a symmetry-breaking transition towards two types of chimera states. Both chimera states inherit properties from the cluster states, from which they originate. In case of type I dynamics, the separation of the two groups is mainly by an amplitude difference, for both clusters and chimeras. Type II chimeras exhibit a phase shift of  $\pi$  between the synchronized and the desynchronized group at the clustering frequency, which is determined by the modulational oscillations. Thus, the two groups are created by the same clustering mechanism that gives also rise to the modulated amplitude clusters. These considerations are in accordance with recent work on the discrete counterparts of the chimera states in the Stuart-Landau ensemble in Ref.<sup>13</sup>, which is reasonable, as we demonstrated that the diffusional coupling is a non-essential ingredient. In fact, most of the dynamics can be understood considering only the nonlinear global coupling, thereby reducing the complexity of the system. As outlined in an earlier work, a bifurcation analysis of the cluster states becomes possible in a two-groups reduction of the Stuart-Landau ensemble<sup>24</sup>. In addition, chimera states are found in an ensemble of Stuart-Landau oscillators with linear global coupling<sup>16</sup> and there is a connection between these chimera states and the type I chimeras in our system: Type I chimeras in the Stuart-Landau ensemble with nonlinear global coupling constitute an idealized case of the chimeras found under linear global coupling<sup>13</sup>. As the discrete dynamics are the essential dynamics, incorporating diffusion yields the same situation, as we demonstrated in the present article. Thus, type I chimeras in the MCGLE and chimera states found in a CGLE with linear global coupling are of the same class. So far, we could not observe modulated amplitude clusters under linear global coupling and consequently, no type II chimera states are found. We suspect that the secondary Hopf bifurcation, leading to modulated amplitude clusters, cannot occur with linear global coupling.

Furthermore, we showed that the MCGLE also exhibits localized turbulence, a phenomenon for which diffusion seems to be necessary. In this state, turbulent spots move in an irregular manner through an otherwise homogeneous system. These spots are randomly created and destroyed. Because of the lack of a phase boundary between the turbulence and the homogeneous regions, we question that the term chimera state is appropriate for the phenomenon of localized turbulence.

In general, a systematic classification of chimera states, as well as a precise definition would be desirable. This requires an understanding of how far it is reasonable to restrict the definition such as: The system separates into two well-defined groups, one synchronized, one incoherent, and each oscillator stays in one group over times

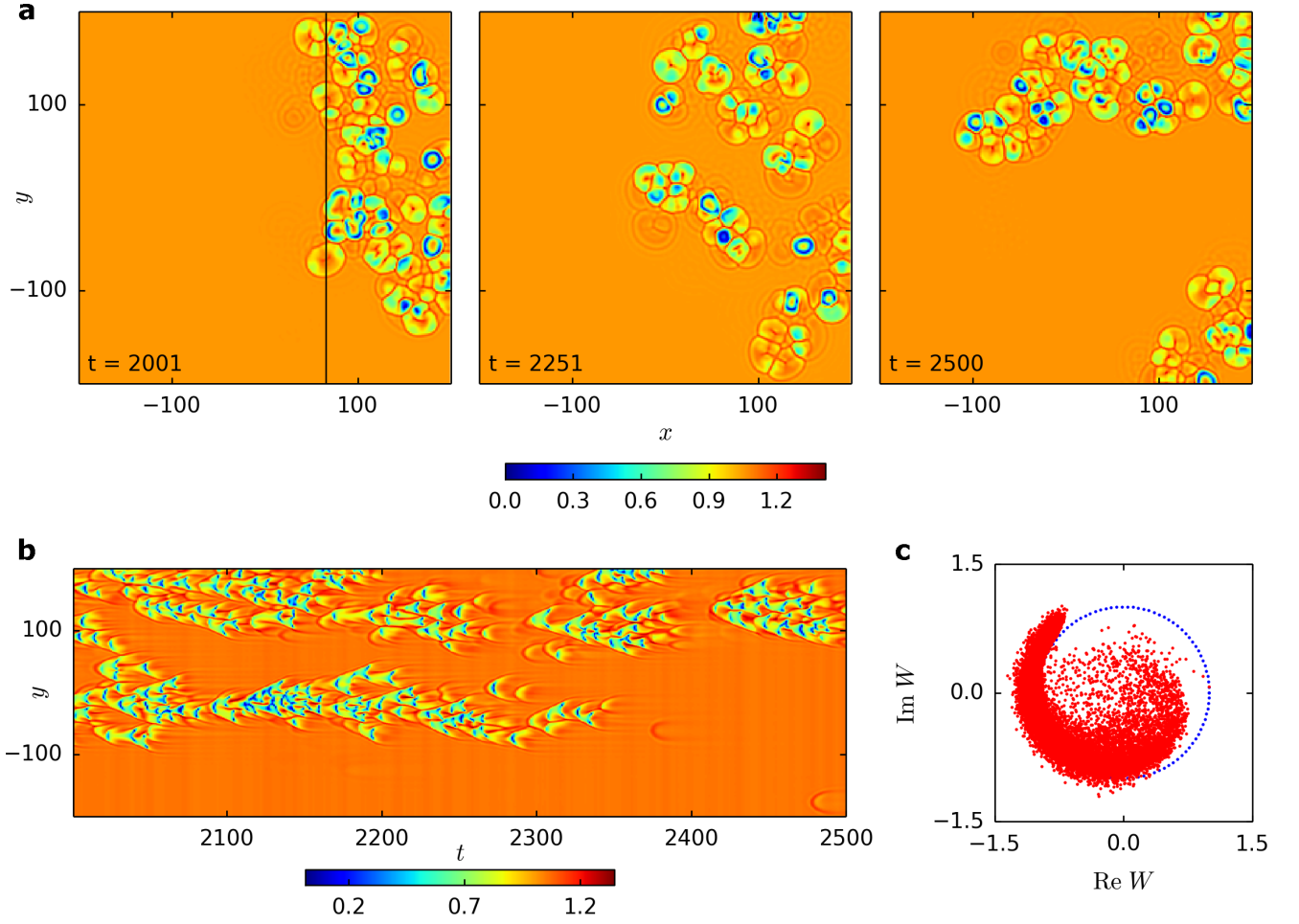


FIG. 7. Localized turbulence in the MCGLE: two-dimensional snapshots of  $|W|$  in (a) at times indicated in the figures, one-dimensional cut (also  $|W|$ ) in (b) as indicated in the first snapshot and snapshot of the arrangement of local oscillators in the complex plane in (c) ( $c_1 = -1.6$ ,  $c_2 = 1.5$ ,  $\nu = 1.5$ ,  $\eta = 0.9$ ).

orders of magnitudes longer than typical oscillation periods. For extended systems one might add that, after initial transients, only oscillators in the boundary change the group. This would provide a profound criterion, determining whether localized turbulence is a chimera state or not.

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## Appendix A

### 1. Simulations of the extended systems

Simulations of Eqs. (1) and (3) in the main text are carried out using a pseudospectral method and an exponential time stepping algorithm<sup>26</sup>. We use  $512 \times 512$  Fourier modes, a computational timestep of  $\Delta t = 0.05$  and system sizes as indicated in the figures. Note that the equation is dimensionless. The system is initialized with a two-dimensional circular perturbation and additional noise. Independent runs with different seeds produce the same qualitative behavior. This even holds, if we consider only random initial conditions without the circular perturbation.

### 2. Simulations of the discrete ensemble

We solved Eqs. (4) using an implicit Adams method with a timestep of  $dt = 0.01$ . Initial conditions are ran-

dom numbers on the real axis fulfilling the conservation law for the mean-field.

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